The simulation testing framework used during the development of OMP-13

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This document details the framework used to simulation test candidate MPs during the development of OMP-13. A summary of assumptions made in this simulation testing framework are listed below, with green highlights indicating updates since de Moor and Butterworth (2012d). Appendix A provides the full details, with data used listed in the tables at the end of the Appendix.

There are two main hypotheses for sardine: a single stock hypothesis or a two-mixing stock hypothesis (with “west” and “south” stocks). Two different types of candidate MPs have been proposed:

a) Candidate MPs which recommend a single directed >14cm sardine TAC and associated ≤14cm sardine bycatch.

b) Candidate MPs which recommend a separate directed >14cm sardine TAC for west and east of Cape Agulhas, and an associated split in the ≤14cm sardine bycatch.

All other sardine bycatches are assumed to be taken from the single or “west” stock only.

There are therefore four alternative possible combinations of sardine TAC/B by area / stock:

i) A single area sardine TAC/B and a single sardine stock.

ii) A two-area sardine TAC/B and a single sardine stock.

iii) A single area sardine TAC/B and two sardine stocks.

iv) A two-area sardine TAC/B and two sardine stocks.

The following assumptions are made in the implementation simulation of Candidate MPs:

i) All sardine catch/bycatch is from the single stock

ii) The TAC/Bs are added and all catch/bycatch is from the single stock

iii) The TAC/Bs are split by stock in a pre-defined proportion, dependent on the ratio of the TAC to the “west” stock 1+ biomass (see de Moor et al. 2013)

iv) The TAC/B for west of Cape Agulhas is assumed taken from the “west” sardine stock and the TAC/B for east of Cape Agulhas is assumed taken from the “south” sardine stock.

Summary list of assumptions made in the framework to be used to simulation test OMP-13

1) Half the sardine is caught between 1 November and 30 April and half from 1 May to 31 October.
2) Half the juvenile anchovy is caught between 1 November and 15 July and half from mid-July to 31 October.

3) Half the adult anchovy is caught between 1 November and 31 March and half from 1 April to 31 October.

4) The assumptions made during the development of the underlying operating models (de Moor and Butterworth 2012b,c,2013a), such as age at maturity and stock-recruitment relationships and differences in these assumptions between alternative operating models (robustness tests), are carried forward during projections.

5) In the underlying operating model which assumes two sardine stocks, the movement of “west” stock recruits to the “south” stock in November is assumed dependent on a relationship with the ratio of the “south” to “west” stock 1+ biomass from the previous November.

6) The recruit survey is simulated to commence mid-May each year.

7) All of the directed (>14cm) sardine \( TAC^S_y \) and \( TAB^S_{big} = 7000t \) (bycatch \( \geq 14cm \)) are 1+ year old sardine.

8) All 1+ sardine catch is split into age groups according to the selectivity-at-age estimated by the underlying operating model.

9) All TABs for <14cm sardine translate into 0-year-old sardine bycatch.

10) All <14cm sardine bycatch with round herring, \( TAB^S_{y,small,rh} = 1000t \), assumed to originate from the single or “west” stock, is caught between the time of the recruit survey (mid-May) and the end of the normal season.

11) Half of the juvenile (<14cm) sardine bycatch with directed sardine is caught by the time of the recruit survey (mid-May), and there is an implicit assumption that all of this bycatch is caught by the end of the normal season (the latter extends to different cut-off dates depending on the assumption made by each MP variant with regards to an additional sub-season).

12) The maximum amount of \( \leq 14cm \) sardine bycatch in the directed (>14cm) sardine catch used to set the sardine TAB, \( \sigma \), is not always assumed taken; a proportion is drawn from a distribution based on the historic proportions with a maximum of \( \sigma \).

13) Half of \( TAB^A = 500t \) is taken by the end of June, with the remaining half taken by the end of the normal season.

14) The initial normal season anchovy TAC, \( TAC^{1,A}_y \), is caught by the end of June, and 65% of this is caught by the end of May with the remaining 35% caught during June.

15) 26% of the anchovy catch landed by the end of June \( (TAC^{1,A}_y + \frac{1}{2}TAB^A) \) are juveniles caught by mid-May.

16) 33% of the anchovy catch landed by the end of June \( (TAC^{1,A}_y + \frac{1}{2}TAB^A) \) are adult anchovy; 69% of the adult anchovy catch is landed by the time of the recruit survey (mid-May).
17) The juvenile (<14cm) sardine bycatch with anchovy is assumed to be taken from the single or “west” sardine stock.

18) The juvenile (<14cm) sardine bycatch with anchovy from January to 31 May is 1.436 times that from January to mid-May.

19) Juvenile (<14cm) sardine bycatch with anchovy over the months of June to December is taken to be a proportion of the anchovy catch during these months, with the monthly proportions and variances being estimated from the monthly juvenile sardine to anchovy ratios, based upon historic catch monthly observations and draws from model predicted recruitment.

20) In the implementation of sardine bycatch with anchovy, correlations in the juvenile single or “west” stock sardine to anchovy ratios apply between successive months only.

21) In the implementation of sardine bycatch with anchovy, if the additional sub-season begins 1 September, 60% of the July and August anchovy catch is taken in July.

22) In the implementation of sardine bycatch with anchovy, if the additional sub-season begins 1 October, 45% of the July to September anchovy catch is taken in July and 30% in August.

23) In the implementation of sardine bycatch with anchovy, if there is no additional sub-season, 42% of the July to December anchovy catch is taken in July, 26% in August and 22% in September.

24) For all catches simulated, an upper limit is placed on the industry’s efficiency by assuming that no more than 95% of the selectivity-weighted stock abundance may be caught.

25) The ratio of juvenile single or “west” stock sardine to anchovy in May (and used in the Harvest Control Rule), \( r_y \), is restricted to a maximum of 1.

26) The ratios of juvenile single or “west” stock sardine to anchovy in the months of June, July, August, September and October to December, used in simulating how much juvenile sardine is actually caught, are restricted to a maximum of 2.

27) The ratio of model predicted November juvenile single or “west” stock sardine to anchovy used when simulating the future single or “west” stock bycatch with anchovy is restricted to a maximum of 1.

28) Sardine bycatch with anchovy in the additional sub-season is at most \( r_y \), the ratio of juvenile single or “west” stock sardine to anchovy “in the sea” during May, of that portion of the anchovy final TAC taken in the additional sub-season.

29) Implementation simulation does not account for the closure of the anchovy fishery if the initial sardine bycatch with anchovy allowance is reached (see de Moor and Butterworth 2012 for reasons), although the sardine bycatch is limited by this allowance.

30) Implementation simulation accounts for the closure of the anchovy fishery if the sardine bycatch with anchovy allowance is reached, by proportionally decreasing the amount of juvenile anchovy catch simulated to be taken within a year.

31) Future survey observations are generated taking the historic correlation between the single or “west” stock sardine and anchovy into account, and the variance is based on a regression between historic survey CV and model predicted abundance.
32) Survey and catch-related observations already known for 2012 have been used instead of model simulated observations. The undercatch of the revised anchovy normal season TAC has been taken into account. The recruitment in November 2011, and the corresponding recruitment residual are obtained by combining information from both the stock recruitment relationship and the known June 2012 survey results.

References


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Appendix A: The framework used to simulation test a joint MP for South African sardine and anchovy: OMP-13

In this appendix, the framework used to simulation test OMP-13 is detailed. The framework consists of a population dynamics model for future simulation of the effects of alternative MPs on the sardine and anchovy populations, an implementation model which generates future catches-at-age given annual TAC/Bs [calculated using the harvest control rules detailed in de Moor and Butterworth 2012a], and an observation model which generates the necessary data (in this case, catch and survey data) to be input into the MP. Catches-at-age are given in numbers of fish (billions), whereas the TACs and TABs are given in biomass (in thousands of tons). All parameters which are drawn from the Bayesian posterior distributions of de Moor and Butterworth (2012b,c) are listed in Table A1.

Population dynamics model

Given the numbers-at-age at the beginning of the projection period [i.e., November 2011, drawn from the posterior distributions output from the operating models (de Moor and Butterworth, 2013b,c,d)], values for future catches output from the implementation model, \( C_{j,y,a}^i \), \( i = S, A \) (see below), the population dynamics model projects numbers-at-age and spawning biomass at the beginning of November for \( y = 2012,\ldots,2032 \) as follows. The sardine adult catch is assumed to be taken half way between 1\textsuperscript{st} November and 31\textsuperscript{st} October each year. (The sardine stock assessment was fit to quarterly commercial proportion at length data and thus catch was modelled to be taken quarterly (de Moor and Butterworth 2012c,2013a). The catch tonnage between 1984 and 2011, however, is almost equally split from 1 November to 30 April and 1 May to 31 October.) The anchovy juvenile catch is assumed to be taken as a pulse at 15\textsuperscript{th} July and the adult catch is assumed to be taken as a pulse at 1\textsuperscript{st} April (de Moor and Butterworth 2012d). Sardine are assumed to mature at age 2 (de Moor and Butterworth 2012c,2013a) and anchovy at age 1 (de Moor and Butterworth 2012d). All notation allows for multiple stocks of both species, though only a single stock for anchovy is considered in all operating models.

Sardine:

\[
N_{j,y,1}^{S,\text{pred}} = \left( N_{j,y-1,0}^{S,\text{pred}} e^{-M_{S_{j,y-1,0}}^{a}/2} - C_{j,y,0}^{S,\text{pred}} \right) e^{-M_{S_{j,y-1,0}}^{a}/2}
\]
\[
N_{j,y,a}^{S,\text{pred}} = \left( N_{j,y-1,a-1}^{S,\text{pred}} e^{-M_{S_{j,y-1,a-1}}^{a}/2} - C_{j,y,a-1}^{S,\text{pred}} \right) e^{-M_{S_{j,y-1,a-1}}^{a}/2}, \quad a = 2,\ldots,4
\]
\[
N_{j,y,5+}^{S,\text{pred}} = \left( N_{j,y-1,4}^{S,\text{pred}} e^{-M_{S_{j,y-1,4}}^{a}/2} - C_{j,y,4}^{S,\text{pred}} \right) e^{-M_{S_{j,y-1,4}}^{a}/2} + \left( N_{j,y-1,5+}^{S,\text{pred}} e^{-M_{S_{j,y-1,5+}}^{a}/2} - C_{j,y,5+}^{S,\text{pred}} \right) e^{-M_{S_{j,y-1,5+}}^{a}/2}
\]
\[
B_{j,y,N}^{S,\text{pred}} = \sum_{a=1}^{5+} N_{j,y,a}^{S,\text{pred}} \bar{w}_{j,a}^{S}
\]
\[
SSB_{j,y,N}^{S,\text{pred}} = \sum_{a=2}^{5+} N_{j,y,a}^{S,\text{pred}} \bar{w}_{j,a}^{S}
\]  
(A.1)

Anchovy:

\[
N_{j,y,1}^{A,\text{pred}} = \left( N_{j,y-1,0}^{A,\text{pred}} e^{-8.5M_{A_{j,y-1,0}}^{a}/12} - C_{j,y,0}^{A,\text{pred}} \right) e^{-3.5M_{A_{j,y-1,0}}^{a}/12}
\]
\[ N^{A,\text{pred}}_{j,y,2} = \left( N^{A,\text{pred}}_{j,y-1,1} e^{-5M^{\text{ad}}_{j,1}/12} - C^{A,\text{pred}}_{j,y,1} \right) e^{-7M^{\text{ad}}_{j,1}/12} \]
\[ N^{A,\text{pred}}_{j,y,3} = N^{A,\text{pred}}_{j,y-1,2} e^{-M^{\text{ad}}_{j,1}} \]
\[ N^{A,\text{pred}}_{j,y,4+} = N^{A,\text{pred}}_{j,y-1,3} e^{-M^{\text{ad}}_{j,1}} + N^{A,\text{pred}}_{j,y-1,4+} e^{-M^{\text{ad}}_{j,1}} \]
\[ B^{A,\text{pred}}_{j,y,N} = \text{SSB}^{A,\text{pred}}_{j,y,N} = \sum_{a=1}^{4} N^{A,\text{pred}}_{j,y,a} \bar{w}^{A}_{j,a} \]

where

\( N^{i,\text{pred}}_{j,y,a} \) is the operating model predicted numbers at age \( a \) (in billions) of species \( i \) (\( i = S, A \)), stock \( j \), at the beginning of November in year \( y \);

\( M^{i}_{ja} \) is the natural mortality rate (in year\(^{-1} \)) of juvenile (age 0) fish of species \( i \) (\( i = S, A \)) (de Moor and Butterworth 2012b,c,2013a);

\( M^{i}_{ad} \) is the natural mortality rate (in year\(^{-1} \)) of age 1+ fish of species \( i \) (\( i = S, A \)) (de Moor and Butterworth 2012b,c,2013a);

\( C^{i,\text{pred}}_{j,y,a} \) is the model predicted future catches at age \( a \) in year \( y \) of species \( i \) (\( i = S, A \)), stock \( j \) output from the implementation model (given below);

\( B^{S,\text{pred}}_{j,y,N} \) is the operating model predicted November 1+ biomass in year \( y \) (in thousands of tons) of species \( i \) (\( i = A, S \)), stock \( j \);

\( \text{SSB}^{i,\text{pred}}_{j,y,N} \) is the operating model predicted spawning stock biomass in year \( y \) (in thousands of tons) of species \( i \) (\( i = A, S \)), stock \( j \); and

\( \bar{w}^{i}_{j,a} \) is the average weights-at-age \( a \) of species \( i \) (\( i = S, A \)), stock \( j \) from the historic November spawner biomass surveys (Table A2).

In the two stock hypothesis, movement of “west” stock (\( j = 1 \)) sardine to the “south” stock (\( j = 2 \)) at the beginning of November, i.e. when the recruits turn age 1, is modelled as a function of the ratio of “south” to “west” stock November 1+ biomass in the previous year (de Moor and Butterworth 2013a), i.e.

\[ \text{move}^*_j = 0.6034 \left( 1 - \exp \left( -2.245 \frac{B^{S,\text{pred}}_{j,y-1,1}}{B^{S,\text{pred}}_{j,y-1,1,N}} \right) \right) \]

In order to allow error about this relationship and satisfy \( 0 \leq \text{move}^*_j \leq 1 \), the logit scale is used. Thus:

\[ \text{move}^*_j = \frac{\exp \left( \ln \left( \frac{\text{move}^*_j}{1 - \text{move}^*_j} \right) + \xi^*_j \right)}{1 + \exp \left( \ln \left( \frac{\text{move}^*_j}{1 - \text{move}^*_j} \right) + \xi^*_j \right)}, \text{ where } \xi^*_j \sim N(0,1.26^2), \text{ with the standard deviation obtained from the model of de Moor and Butterworth (2013a), corrected for bias. Then} \]
\[
N_{1,y,1}^{\text{pred}} = (1 - \text{move}) N_{1,y,1}^{\text{prev}}
\]

\[
N_{2,y,1}^{\text{pred}} = N_{2,y,1}^{\text{prev}} + \text{move} N_{1,y,1}^{\text{prev}}
\]

\[y = y_1, \ldots, y_n \]

(A.14)

where \(N_{1,y,1}^{\text{prev}}\) is simply the numbers-at-age 1 given by equation (A.1) prior to movement.

Letting \(f(\text{SSB}_{i,j,N}^{\text{pred}})\) denote the stock recruitment curve of the chosen model, with parameters \(a_i^j\) and \(b_i^j\) (Table A1), then future recruitment \(N_{i,j,0}^{\text{pred}}\) \((i = S, A)\) is assumed to be log-normally distributed about a stock recruitment relationship as follows:

\[
N_{i,j,0}^{\text{pred}} = f(\text{SSB}_{i,j,N}^{\text{pred}}) e^{\epsilon_{i,j}^r} \sigma_{i,j}^r
\]

(A.3)

where

\[
\epsilon_{i,j}^r = s_{j,\text{cor}}^i \epsilon_{i,j-1} + \sqrt{1 - (s_{j,\text{cor}}^i)^2} \omega_{i,j}^r, \text{ where } \omega_{i,j}^r \sim N(0;1)
\]

(A.4)

and

\(\epsilon_{i,j}^r\) is the standardised recruitment residual for stock \(j\) of species \(i\) \((i = S, A)\) in year \(y\), see below for \(\epsilon_{i,j}^{r,2011}\);

\(\sigma_{i,j}^r\) is the standard deviation of the recruitment residuals for stock \(j\) of species \(i\) \((i = S, A)\), drawn from posterior distributions output from the operating models (de Moor and Butterworth, 2013b,c,d); and

\(s_{j,\text{cor}}^i\) is the recruitment serial correlation for stock \(j\) of species \(i\) \((i = S, A)\), drawn from posterior distributions output from the operating models (de Moor and Butterworth, 2013b,c,d).

Implementation model

The MP variants outputs the following TAC/Bs (see de Moor and Butterworth 2012a):

1) An annual directed \(>14\text{cm sardine TAC}, TAC_y^{S}\), which may be split by area (\(TAC_y^{S,w}\) and \(TAC_y^{S,e}\)) in a candidate MP which allocates sardine TAC west and east of Cape Agulhas.

2) An initial and revised normal season anchovy TAC (\(TAC_y^{A}\) and \(TAC_y^{A,2}\)), either applying to i) January to 31 August, ii) January to 30 September, or iii) January to December depending on the assumptions regarding the additional sub-season in the MP variant being tested. In cases i) and ii), where an additional sub-season is included from September/October to December, a total anchovy TAC, which includes the additional sub-season with the normal season, is given by \(TAC_y^{A,3}\).

3) An annual constant anchovy TAB, \(TAB_y^{A}\).

4) An annual constant \(>14\text{cm sardine TAB}, TAB_{big}^{S}\).

5) An annual constant \(\leq 14\text{cm sardine bycatch with round herring, and to a lesser extent with anchovy, TAB}_{\text{small},\text{rh}}^{S}\).
6) An annual ≤14cm sardine bycatch with directed (>14cm) sardine, $TAB_{y,small}^{S}$, which may be split by area ($TAB_{y,small}^{S,w}$ and $TAB_{y,small}^{S,e}$) in a candidate MP which allocates sardine TAC\textsuperscript{1} west and east of Cape Agulhas.

7) For each anchovy TAC in 2) there is a corresponding ≤14cm sardine TAB with anchovy, $TAB_{y,anch}^{S}$, $TAB_{y,anch}^{S,1}$, and, when applicable, $TAB_{y,anch}^{S,2}$.

Given these TAC / TABs output from the MP (in thousands of tons), the implementation model simulates the implementation of these catch limits by the industry to yield future catches-at-age (in billions).

There are four alternative possible combinations of sardine TAC/B by area / stock:

i) A single area sardine TAC/B and a single sardine stock.

ii) A two-area sardine TAC/B and a single sardine stock.

iii) A single area sardine TAC/B and two sardine stocks.

iv) A two-area sardine TAC/B and two sardine stocks.

Defining $TAC_{y,j}^{S}$ to be the directed >14cm sardine TAC assumed taken from stock $j$, the following separation of TAC by stock is effected:

i) $TAC_{y,1}^{S} = TAC_{y,1}^{S,1}$ and $TAB_{y,small}^{S} = TAB_{y,small}^{S,1}$, with only a single sardine stock

ii) $TAC_{y,1}^{S} = TAC_{y,1}^{S,w} + TAC_{y,1}^{S,e}$ and $TAB_{y,small}^{S} = TAB_{y,small}^{S,w} + TAB_{y,small}^{S,e}$ with only a single sardine stock

iii) $TAC_{y,1}^{S} = \tau_{1}TAC_{y,1}^{S}$ and $TAC_{y,2}^{S} = \tau_{2}TAC_{y,2}^{S}$, and $TAB_{y,small}^{S} = \tau_{1}TAB_{y,small}^{S,1}$ and $\tau_{2}TAB_{y,small}^{S,2}$

iv) $TAC_{y,1}^{S} = TAC_{y,1}^{S,w}$ and $TAC_{y,2}^{S} = TAC_{y,2}^{S,e}$, and $TAB_{y,small}^{S} = TAB_{y,small}^{S,w}$ and $TAB_{y,small}^{S,e}$

where

$\tau_{j}$ is the proportion of the directed >14cm sardine TAC assumed caught west ( $j = 1$ ) / south ( $j = 2$ ) of Cape Agulhas. The ≤14cm sardine bycatch with directed (>14cm) sardine is assumed to be proportioned west/south of Cape Agulhas in the manner as the TAC. Note $\tau_{1} + \tau_{2} = 1$.

The annual proportion of sardine catch taken west Cape Agulhas was found to have a relationship with the ratio of the TAC in a particular year to the “west” stock 1+ biomass in November of the previous year (de Moor \textit{et al}. 2013). Thus we have

\footnote{This TAB is closely related to the directed catch corresponding to the area-specific TACs.}
\[ \tau_i^* = 1 - 0.8864 + \frac{TAC^S_y / B_{1,y-1}^S}{0.8905 + TAC^S_y / B_{1,y-1}^S}, \]

In order to allow errors about this relationship and satisfy \( 0 \leq \tau_i \leq 1 \), the logit scale is used. Thus:

\[ \tau_i = \frac{\exp \left( \ln \left( \frac{\tau_i^*}{1 - \tau_i^*} \right) + \xi_y^* \right)}{1 + \exp \left( \ln \left( \frac{\tau_i^*}{1 - \tau_i^*} \right) + \xi_y^* \right)}, \]

where \( \xi_y^* \sim N(0,0.78^2) \), with the standard deviation obtained from the model of de Moor et al. (2013), corrected for bias.

### Sardine adult catch

The adult sardine catch is simulated using selectivity-at-age estimated by the operating model:

\[ C_{j,y,a}^{S,\text{pred}} = N_{j,y-1,a}^{S,\text{pred}} S_{j,a}^S F_{j,y} e^{-M_a^y/2}, \quad a = 1, ..., 5 + \]  

(A.5)

where

\[ F_{j,y} = \frac{TAC_{j,y}^S + \tau_y^* \text{TAB}_{\text{big}}^S}{\sum_{a=1}^{5+} N_{j,y-1,a}^{S,\text{pred}} S_{j,a}^S \bar{w}_{j,ac}^i e^{-M_a^y/2}}, \]  

(A.6)

and

\[ S_{j,a}^S \] are the sardine stock \( j \) fishing selectivities-at-age \( a \) drawn from posterior distributions estimated from the operating model (de Moor and Butterworth 2013c,d);

\[ \bar{w}_{j,ac}^i \] are the historic average weights-at-age \( a \) in the catches for stock \( j \) of species \( i \), \( i = A,S \) (Table A2);

\( \tau_y^* \) is the proportion of the big sardine TAB assumed caught west/south of Cape Agulhas. For OMP-13 the assumption is that \( \tau_1^* = 1 \) and \( \tau_2^* = 0 \).

### Anchovy 1-year-old catch

Between 1984 and 2011, the total (annual) 1-year-old catch in tons constituted, on average, 37% of the anchovy catch biomass between January and June (the period to which \( TAC_{y}^{A,1} \) and half of \( \text{TAB}^A \) is taken to apply). This percentage drops to 33% if only the past 10 years are considered. As the most recent history is likely a better reflection of future catch patterns, the anchovy 1 year old catch is thus taken to be 33% of the initial normal season anchovy TAC:

\[ C_{1,y,1}^{A,\text{pred}} = 0.33 \times \frac{TAC_{y}^{1,A} + \frac{1}{2} \text{TAB}^A}{\bar{w}_{1c}^A}. \]  

(A.7)

\[ ^2 \text{Taken to be the average of model predicted commercial selectivity-at-age in quarters 2 and 3 of de Moor and Butterworth (2012c,2013a). The selectivities-at-ages 1 to 5+ are re-normalised such that the largest selectivity is 1.} \]
Anchovy 0-year-old catch

Between 1984 and 2011 the anchovy juvenile catch in tons from 1st January to 30th April, together with half the May juvenile catch in tons was 26% of the total anchovy catch biomass from January to June. This percentage remains unchanged if only the past 10 years are considered. Using the above assumption that \( TAC^A_y \) and half of \( TAB^A \) is caught by the end of June, the anchovy 0-year-old catch taken prior to the recruit survey is:

\[
C^{A, pred}_{1,y,0} = 0.26 \left( \frac{TAC^{1,A}_y + \frac{1}{2} TAB^A}{W_{0c}} \right) \tag{A.8}
\]

and for the normal season as a whole:

\[
C^{A*, pred}_{1,y,0} = \frac{1}{W_{0c}} \left( TAC^{2,A}_y + TAB^A - C^{A, pred}_{1,y,1} \times W_{1c} \right) \tag{A.9}
\]

Sardine 0-year-old catch prior to the recruit survey

The 0-year-old sardine catch prior to the recruit survey is based on the January to mid-May bycatch occurring with i) round herring, ii) adult sardine in the directed fishery, and iii) targeted juvenile anchovy. It is assumed that all juvenile sardine bycatch with round herring occurs after the recruit survey. It is further assumed that half the juvenile sardine in the directed sardine catch is caught by the time of the survey:

\[
C^{S, pred}_{1,y,0} = \frac{1}{2} \frac{\sigma_{\text{draw}}^{y} TAC^{1,S}_{1,y}}{W_{1,0c}} + k_{\text{jan, may}} \frac{N^{y, pred}_{1,y,1} - 1.0}{N^{y, pred}_{1,y,1.0}} e^{\sigma_{\text{draw}}^{y} \eta_{\text{jan, may}}} \frac{0.26 \times TAC^{1,A}_y}{W_{1,0c}},
\]

\[
C^{S, pred}_{2,y,0} = \frac{1}{2} \frac{\sigma_{\text{draw}}^{y} TAC^{2,S}_{2,y}}{W_{2,0c}}.
\]

where \( \eta_{y, \text{jan, may}} \sim N(0;1) \) \tag{A.10}

and \( k_{\text{jan, may}} \) and \( \sigma_{\text{jan, may}} \) are given in equations (A.41) and (A.43) respectively. \( \sigma \) is the estimate of the maximum amount of \( \leq 14 \text{cm} \) sardine bycatch in the directed (>14cm) sardine catch used to set the sardine TAB. During simulation, this maximum amount is not always assumed taken. Instead, the proportion, \( \sigma_{\text{draw}}^{y} \), of the directed catch assumed taken is drawn from a distribution based on the historic proportions (Figure A1).

Sardine 0-year-old catch (in billions)

In modelling the total sardine juvenile bycatch, the following approach is used. If the full TAB with anchovy were caught, the total juvenile sardine catch by mass would be

\[
\frac{W_{1,0c}}{2} C^{S, pred}_{1,y,0} = (\lambda, TAC^{1,A}_y + r_{1} (TAC^{2,A}_y - TAC^{1,A}_y) + \tau_{TAB}^{S, small, ch} + \omega^{\text{draw}}_{y} TAC^{S}_{1,y}),
\]

\[
\frac{W_{2,0c}}{2} C^{S, pred}_{2,y,0} = \tau_{2}^{TAB} TAC^{S}_{2,y} + \omega^{\text{draw}}_{y} TAC^{S}_{2,y}.
\]
where \( \lambda_y = \max \{ \gamma_y, r_j \} \) \hspace{1cm} (A.11)\n
\( \gamma_y \) is the percentage of the initial anchovy TAC used to set the initial \( \leq 14 \text{cm} \) sardine TAB with anchovy. We now need to model how the sardine bycatch with anchovy fishery develops during the season; and \( \tau''_j \) is the proportion of the small sardine TAB with redeye assumed caught west/south of Cape Agulhas.

For OMP-13 the assumption is that \( \tau''_1 = 1 \) and \( \tau''_0 = 0 \).

The ratio of juvenile sardine to anchovy “in the sea” during May, \( r_y \), is calculated from two sources as follows:

\[ r_y = \frac{1}{2} (r_{y,sur} + r_{y,com}) \] \hspace{1cm} (A.12)\n
When implementing OMP-13, both \( r_{y,sur} \) and \( r_{y,com} \) will be observations that will be available to input into the Harvest Control Rules, with

\[ r_{y,sur} = \frac{N_{s,obs}^{y}}{N_{A,obs}^{y}} \] \hspace{1cm} (A.13)\n
During simulation, the sardine bycatch to anchovy ratio in commercial catches in May, is given by:

\[ r_{y,com} = k_{may} \frac{N_{s,pred}^{y}}{N_{A,pred}^{y}} e^{\sigma_{may} e_{y,may}} \] \hspace{1cm} (A.14)\n
where \( e_{y,may} = \rho_{may} \eta_{y,jan,may} + \sqrt{1 - (\rho_{may})^2} \eta_{y,may} \) \hspace{1cm} (A.15)\n
with \( \eta_{y,may} \sim N(0;1) \) and \( \eta_{y,jan,may} \) is given by equation (A.10). As \( r_{y,com} \) is based on simulated commercial catches, the model predicted numbers-at-age, \( N_{i,pred}^{y,j,y} \), are used rather than those simulated to be survey observations. Here we have

\( N_{i,obs}^{y,j,y} \) - the acoustic survey estimate of recruitment (in billions) for fish of stock \( j \) of species \( i \) (\( i = S, A \)) for year \( y \), which will be an observation available for input into the Harvest Control Rules; during simulation these observations are derived using equation (A.31).

\( N_{i,pred}^{y,j,y} \) - the model-predicted recruitment (in billions) for fish of stock \( j \) of species \( i \) (\( i = S, A \)) in November of year \( y-1 \), projected forward to the time of the recruit survey in year \( y \) (equation A.34).

\( k_{may} \) - the constant of proportionality from equation (A.44),

\( \sigma_{may} \) - the residual standard deviation from equation (A.46); and

\( \rho_{may} \) - the correlation coefficient from equation (A.47)

Equation (A.11) assumes that the ratio of juvenile sardine to anchovy “in the sea” during May, \( r_y \), will remain a constant for the remainder of the normal season. However, there is usually a drop-off in this ratio as the year
progresses (Figure A2). This effect is simulated by adjusting equation (A.11) to reflect the actual level of 0-year-old sardine to be expected in the catches, given the historical pattern of sardine bycatch to anchovy ratio changes (usually a drop-off) from May to August/September/October-December\(^3\).

Over the past 10 years (2002-2011), the sardine bycatch with anchovy from January to 31\(^{st}\) May has been 1.436 times that from January to mid-May\(^4\). Adjusting the sardine bycatch prior to the survey to take account of this additional bycatch by the end of May, the catch from the “west” or single stock in equation (A.11) is modified as follows:

i) If the MP variant assumes the additional sub-season runs from 1 September to 31 December:

\[
C_{l,y,0}^{S,pred} = 1.436 \times \left( C_{l,y,0Hu}^{S,pred} + \frac{1}{2} \frac{\sigma_{y}^{draw} TAC_{l,y}^{S} + \sigma_{y}^{draw} TAC_{l,y}^{S}}{W_{1,0c}} \right) + \frac{1}{W_{1,0c}} \left( r_{y,jun} A_{y,jun}^{A,pred} + r_{y,jul} A_{y,jul}^{A,pred} + r_{y,aug} A_{y,aug}^{A,pred} \right)
\]

(A.16a)

ii) If the MP variant assumes the additional sub-season runs from 1 October to 31 December:

\[
C_{l,y,0}^{S,pred} = 1.436 \times \left( C_{l,y,0Hu}^{S,pred} + \frac{1}{2} \frac{\sigma_{y}^{draw} TAC_{l,y}^{S} + \sigma_{y}^{draw} TAC_{l,y}^{S}}{W_{1,0c}} \right) + \frac{1}{W_{1,0c}} \left( r_{y,jun} A_{y,jun}^{A,pred} + r_{y,jul} A_{y,jul}^{A,pred} + r_{y,aug} A_{y,aug}^{A,pred} + r_{y,sep} A_{y,sep}^{A,pred} \right)
\]

(A.16b)

iii) If the MP variant assumes no additional sub-season:

\[
C_{l,y,0}^{S,pred} = 1.436 \times \left( C_{l,y,0Hu}^{S,pred} + \frac{1}{2} \frac{\sigma_{y}^{draw} TAC_{l,y}^{S} + \sigma_{y}^{draw} TAC_{l,y}^{S}}{W_{1,0c}} \right) + \frac{1}{W_{1,0c}} \left( r_{y,jun} A_{y,jun}^{A,pred} + r_{y,jul} A_{y,jul}^{A,pred} + r_{y,aug} A_{y,aug}^{A,pred} + r_{y,sep} A_{y,sep}^{A,pred} + r_{y,octdec} A_{y,octdec}^{A,pred} \right)
\]

(A.16c)

The sardine bycatch to anchovy ratios, \(r_{y,m}\), are simulated in a similar way to \(r_{y,com}\) (equation A.14) as follows:

\[
r_{y,m} = k_{m} \frac{N_{1,y,r}^{S,pred} e_{\sigma_{m}} e_{s,m}}{N_{1,y,r}^{A,pred}}, \text{ where } m = \text{jun, jul, aug, sep, octdec}
\]

(A.17)

where \(k_{m}\) and \(\sigma_{m}\) are from equations (A.44) and (A.46), summing over years for which anchovy directed catch is non-zero, and:

\[
e_{y,jun} = \rho_{y,jun} e_{y,may} + \sqrt{1 - (\rho_{y,jun})^2} \eta_{y,jun}
\]

\(^3\) The choice of month depends on the end of the normal season in the MP variant.

\(^4\) Bycatch from 1\(^{st}\) to 15\(^{th}\) May approximated by half the bycatch from the full month of May.
\[ \varepsilon_{y, jul} = \rho_{y, jul} \varepsilon_{y, jun} + \sqrt{1 - (\rho_{y, jul})^2} \eta_{y, jul} \]
\[ \varepsilon_{y, aug} = \rho_{y, aug} \varepsilon_{y, jul} + \sqrt{1 - (\rho_{y, aug})^2} \eta_{y, aug} \]
\[ \varepsilon_{y, sep} = \rho_{y, sep} \varepsilon_{y, aug} + \sqrt{1 - (\rho_{y, sep})^2} \eta_{y, sep} \]
\[ \varepsilon_{y, octdec} = \rho_{y, octdec} \varepsilon_{y, sep} + \sqrt{1 - (\rho_{y, octdec})^2} \eta_{y, octdec} \]  
(A.18)

The equations above reflect the correlative relationships between adjacent months, where \( \varepsilon_{y,may} \) is from equation (A.15), \( \rho_m \) is from equation (A.47) and \( \eta_{y,m} \sim N(0;1), m = \text{jun, jul, aug, sep, octdec} \).  
(A.19)

Between 2002 and 2011 the average total anchovy catch from January to May was 65\(^{6}\)% of that from January to June.  Assuming 65\(^{6}\)\% of \( TAC_{y}^{1,A} \) is caught by the end of May, and given the assumption that \( TAC_{y}^{1,A} \) is caught by the end of June, the anchovy catches in equation (A.16), \( A_{y,m}^{A,\text{pred}} \) \( m = \text{jun, jul, aug, sep} \), are derived as follows (in thousands of tons):
\[ C_{y, jul}^{A,\text{pred}} = 0.35 \times TAC_{y}^{1,A} \]  
(A.20)
\[ C_{y, jul}^{A,\text{pred}} = p_{y, jul} \left( TAC_{y}^{2,A} - TAC_{y}^{1,A} \right) \]  
(A.21)
\[ \text{i) For the case where a MP variant assumes the additional sub-season begins 1 September:} \]
\[ C_{y, aug}^{A,\text{pred}} = \left( 1 - p_{y, jul} \right) \left( TAC_{y}^{2,A} - TAC_{y}^{1,A} \right) \]  
(A.22a)
where \( p_{y, jul} = 0.60^{7} \) is taken to be the average 2002 to 2011 proportion of total anchovy catch during July and August that is taken in July.
\[ \text{ii) For the case where a MP variant assumes the additional sub-season begins on 1 October:} \]
\[ C_{y, aug}^{A,\text{pred}} = p_{y, aug} \left( TAC_{y}^{2,A} - TAC_{y}^{1,A} \right) \]  
(A.22b)
\[ C_{y, sep}^{A,\text{pred}} = \left( 1 - p_{y, jul} - p_{y, aug} \right) \left( TAC_{y}^{2,A} - TAC_{y}^{1,A} \right) \]  
(A.23b)
where \( p_{y, jul} = 0.45^{8} \) and \( p_{y, aug} = 0.30^{9} \) are taken to be the average 2002 to 2011 proportion of total anchovy catch during July to September that is taken in July and August, respectively.
\[ \text{iii) For the case where a MP variant assumes no additional sub-season:} \]
\[ C_{y, aug}^{A,\text{pred}} = p_{y, aug} \left( TAC_{y}^{2,A} - TAC_{y}^{1,A} \right) \]  
(A.22c)
\[ C_{y, sep}^{A,\text{pred}} = p_{y, sep} \left( TAC_{y}^{2,A} - TAC_{y}^{1,A} \right) \]  
(A.23c)
\[ C_{y, octdec}^{A,\text{pred}} = \left( 1 - p_{y, jul} - p_{y, aug} - p_{y, sep} \right) \left( TAC_{y}^{2,A} - TAC_{y}^{1,A} \right) \]  
(A.24c)

\(^{5}\) Note that \( \varepsilon_{y,m-1}' \) is replaced by \( \varepsilon_{y,m-1}'' \) in the numerator of equation (A.47) for \( m = \text{jun, jul, aug, sep} \).
\(^{6}\) Average from 1984 to 2011 is 70\%.
\(^{7}\) Average from 1984 to 2011 is 0.59.
\(^{8}\) Average from 1984 to 2011 is 0.49.
\(^{9}\) Average from 1984 to 2011 is 0.30.
where \( p_{ja} = 0.42^{10} \), \( p_{aug} = 0.26^{11} \) and \( p_{sep} = 0.22^{12} \) are taken to be the average 2002 to 2011 proportion of total anchovy catch during July to December that is taken in July, August and September, respectively.

**Closure of the anchovy fishery**

The anchovy catch, \( C_{1,y,0}^A \), is adjusted if the adjusted \( C_{1,y,0}^S \) exceeds \( T A B_{y,anch}^{2,S} \) (equation (OMP.12) of de Moor and Butterworth 2012a), in order to reflect the closure of the anchovy fishery once the sardine bycatch allowance linked to anchovy is reached. If \( C_{1,y,0}^{S+,pred} \cdot W_{1,0c}^S > T A B_{y,anch}^{2,S} \), then the anchovy fishery would be closed once the full bycatch allowance was taken. This is simulated by assuming that the anchovy TAC is taken at the same rate as the sardine bycatch:

\[
C_{1,y,0}^{S+,pred} = \min \left\{ C_{1,y,0}^{S+,pred}, \frac{T A B_{y,anch}^{2,S}}{W_{1,0c}^S} \right\}
\]  

(A.24)

\[
C_{1,y,0}^{A+,pred} = \begin{cases} 
\frac{1}{W_{0c}^A} \left[ T A C_{y}^{2,A} \left( \frac{T A B_{y,anch}^{2,S}}{C_{1,y,0}^{S+,pred} \cdot W_{0c}^S} \right) + T A B - C_{1,y,1}^{A+,pred} \cdot W_{1c}^A \right] & \text{if } C_{1,y,0}^{S+,pred} \cdot W_{1,0c}^S \leq T A B_{y,anch}^{2,S} \\
\frac{1}{W_{0c}^A} \left( T A C_{y}^{2,A} - T A C_{y}^{3,A} \right) & \text{if } C_{1,y,0}^{S+,pred} \cdot W_{1,0c}^S > T A B_{y,anch}^{2,S}
\end{cases}
\]  

(A.25)

**Additional sub-season**

A final adjustment is made to \( C_{1,y,0}^{S+,pred} \) and \( C_{1,y,0}^{A+,pred} \) as given by equations (A.24) and (A.25), only for the MP variants that assume an additional sub-season, to reflect the catches taken in the additional sub-season, as follows:

\[
C_{1,y,0}^{S,\text{pred}} = C_{1,y,0}^{S+,\text{pred}} + \frac{1}{W_{1,0c}^S} \min \left\{ T A B_{\text{adj}}^S, r_y \left( T A C_{y}^{3,A} - T A C_{y}^{2,A} \right) \right\}
\]  

(A.26)

\[
C_{1,y,0}^{A,\text{pred}} = C_{1,y,0}^{A+,\text{pred}} + \frac{1}{W_{0c}^A} \left( T A C_{y}^{3,A} - T A C_{y}^{2,A} \right)
\]  

(A.27)

which ensures that the bycatch in the additional sub-season is at most \( r_y \) of that portion of the anchovy final TAC taken in the sub-season.

**General**

For all catches simulated in the operating model, an upper limit is placed on the industry’s efficiency by assuming that no more than 95% of the selectivity-weighted stock abundance may be caught. Furthermore, appropriate adjustments are made to ensure non-negative values for catches.

---

10 Average from 1984 to 2011 is 0.44.
11 Average from 1984 to 2011 is 0.28.
12 Average from 1984 to 2011 is 0.18.
Observation Model

The survey estimates for spawner biomass and recruitment are generated by the observation model as follows:

\[ B_{i,\text{obs}}^{j,y,N} = k_{j,N}^i B_{j,N}^{i,\text{pred}} e^{\epsilon_{i,y,N}} \]

where \( \epsilon_{i,y,N} = \eta_{i,y,N}^S \tilde{\sigma}_{i,y,N}^S \), where \( \eta_{i,y,N}^S \sim N(0;1) \)

and \( \epsilon_{i,y,N}^A = \rho_{Nov}^i \eta_{i,y,N}^S + \sqrt{1 - (\rho_{Nov}^i)^2} \eta_{i,y,N}^A \tilde{\sigma}_{i,y,N}^A \), where \( \eta_{i,y,N}^A \sim N(0;1) \)

For a single sardine stock OM:

\[ \tilde{\sigma}_{i,y,N}^S = \min \left( 1.1181^2, 0.000 + \frac{136.6338}{B_{i,\text{pred}}^{S,j,y,N}} \right) + (\varphi_{sc}^S)^2 + (\lambda_{i,N}^S)^2 \]

For a two sardine stock OM:

\[ \tilde{\sigma}_{2,y,N}^S = \min \left( 1.2293^2, 0.407 + \frac{0.0083}{B_{2,\text{pred}}^{S,j,y,N}} \right) + (\varphi_{ac}^S)^2 + (\lambda_{i,N}^S)^2 \]

and \( \tilde{\sigma}_{i,y,N}^A = \min \left( 0.4096^2, 0.0242 + \frac{11.1157}{B_{A,\text{pred}}^{i,j,y,N}} \right) + (\lambda_{i,N}^A)^2 \)

obtained from a regression of the observed CV against the base case assessment model predicted biomass between 1984 and 2011 (Figure A3). Here

- \( B_{i,\text{obs}}^{j,y,N} \) is the November acoustic survey estimate of stock \( j \) of species \( i (i = A, S) \) 1+ biomass (in thousands of tons) in year \( y \);
- \( k_{j,N}^i \) is the constant of proportionality (multiplicative bias) between survey estimated and model predicted 1+ biomass of stock \( j \) of species \( i (i = S, A) \), drawn from posterior distributions output from the operating models (de Moor and Butterworth, 2013b,c,d);
- \( \rho_{Nov}^i \) is the correlation in the residuals between the sardine and anchovy November survey estimates, given by equations (A.37);
- \( \varphi_{sc}^S \) is the CV associated with the factors which cause bias in the sardine acoustic survey estimates and which vary inter-annually rather than remain fixed over time (de Moor and Butterworth 2012c,2013a);
- \( (\lambda_{i,N}^S)^2 \) is the additional variance (over and above the squares of the survey sampling CV and of the CV

\[13\] In the two sardine stock hypothesis, the assumption is made that anchovy biomass and recruitment is only correlated with the “west” stock.
\[14\] From the sardine single stock base case hypothesis with hockey stick stock recruitment curve
\[15\] From the sardine two stock base case hypothesis with hockey stick stock recruitment curves.
\[16\] From the anchovy base case assessment assuming a Beverton Holt stock recruitment curve.
\( \phi_{\text{sac}}^S \) associated with the November/recruit surveys of stock \( j \) of species \( i \) \((i = S, A)\); and

\[
N_{j,i,\text{obs}}^{i,j} = k_{j,i} N_{j,i,\text{pred}}^{i,j} e^{e_{j,i,\text{rec}}} ,
\]

where \( e_{j,i,\text{rec}} = \eta_{j,i,\text{rec}}^S \sigma_{j,i,\text{rec}}^S \), and

\[
e_{j,i,\text{rec}}^A = \left( \rho_{\text{rec}} \eta_{j,i,\text{rec}}^A \right) + \sqrt{1 - \left( \rho_{\text{rec}} \right)^2} \eta_{j,i,\text{rec}}^A \sigma_{j,i,\text{rec}}^{A,13},
\]

where \( \eta_{j,i,\text{rec}}^A \sim N(0;1) \).

For a single sardine stock OM:

\[
\bar{\sigma}_{1,y,\text{rec}}^S = \sqrt{\min \left( 1.0785^2, 0.0987 + \frac{1.5010}{N_{1,y,\text{pred}}^{S,\text{pred}}} \right) + \left( \phi_{\text{rec}}^S \right)^2 + \left( \lambda_{1,y}^S \right)^2}^{14}
\]

For a two sardine stock OM:

\[
\bar{\sigma}_{2,y,\text{rec}}^S = \sqrt{\min \left( 1.0184^2, 0.4767 + \frac{0.5017}{N_{2,y,\text{pred}}^{S,\text{pred}}} \right) + \left( \phi_{\text{rec}}^S \right)^2 + \left( \lambda_{2,y}^S \right)^2}^{15}
\]

and

\[
\bar{\sigma}_{1,y,\text{rec}}^A = \sqrt{\min \left( 0.2830^2, 0.0372 + \frac{0.0}{N_{1,y,\text{pred}}^{A,\text{pred}}} \right) + \left( \phi_{\text{rec}}^A \right)^2 + \left( \lambda_{1,y}^A \right)^2}^{16}
\]

obtained from a regression of the observed CV against the base case assessment model predicted recruitment between 1985 and 2011 (Figure A3).

Here

\( k_{j,i} \) is the constant of proportionality (multiplicative bias) between survey estimated and model predicted 1+ biomass, drawn from posterior distributions estimated by the operating models (de Moor and Butterworth, 2012b,c); and

\( \rho_{\text{rec}} \) is the correlation in the residuals between the sardine and anchovy recruit survey estimates, given by equation (A.40).

Assuming that the recruit survey begins mid-May each year, and that juvenile sardine are caught half-way between 1 November and the start of the survey, while juvenile anchovy caught prior to the survey are taken in a pulse at 1 May (in line with the assumptions made in de Moor and Butterworth 2012b,c,2013a), we simulate:

\[
N_{j,i,\text{pred}}^{S,\text{pred}} = \left( N_{j,i,\text{obs}}^{S,\text{obs}} e^{-3.25M_{\mu}^{S,12}} - C_{j,i,\text{rec}}^{S,\text{pred}} e^{-3.25M_{\mu}^{S,12}} \right)
\]

\[
N_{j,i,\text{pred}}^{A,\text{pred}} = \left( N_{j,i,\text{obs}}^{A,\text{obs}} e^{-0.5M_{\mu}^{A,12}} - C_{j,i,\text{rec}}^{A,\text{pred}} e^{-0.5M_{\mu}^{A,12}} \right).
\]

Assumptions made for 2011 and 2012

As the stock assessments (de Moor and Butterworth 2013b,c,d) covered the period to November 2011, the MP testing framework begins from November 2011 and projects to November 2032. A number of parameters that would be simulated in the testing framework for 2012, have however already been observed. Thus the following changes are made to the simulation framework above for 2011/2012:
i) The TAC/TABs (in thousands of tons) for 2012 have already been set using OMP-08, thus

\[ TAC^S_{2012} = 100.595, \quad TAC^{1A}_{2012} = 202.718, \quad TAB^{1S}_{2012,anch} = 25.4466 - 3.5 = 21.9466, \]

\[ TAC^{2A}_{2012} = 352.718, \quad TAB^{2S}_{2012,anch} = 35.8791 - 3.5 = 32.3791, \]

\[ TAC^{3A}_{2012} = TAC^{2A}_{2012} + 120, \quad TAB^{3S}_{2012,anch} = TAB^{2S}_{2012,anch} + 2 \]

For candidate MPs which calculate area-specific sardine TAC/Bs, the assumption is made that the TAC was awarded in the same proportion as assumed in a single area/two stock scenario (see page 9 above). Thus

ii) The anchovy catch from 1st January to 30th June 2012 closely resembles \( TAC^{1A}_{2012} \). However, the anchovy catch from 1st July to 31st August 2012 of about 66 thousand tons (van der Westhuizen pers. comm.) is much less than \( TAC^{2A}_{2012} - TAC^{1A}_{2012} = 150 \). Thus during implementation (equations (A.9), (A.21)-(A.27)), \( TAC^{2A}_{2012} \) is replaced with 268.5 thousand tons and \( TAC^{3A}_{2012} \) is replaced with 388.5 thousand tons.

iii) As the May 2012 survey observations are available, no error is required, thus equation (A.31) is replaced by \( N_{obs,S}^{12,1} = 8.103 \) billion for either the single stock OM or the “west” stock of the two stock OM, and \( N_{obs,A}^{12,1} = 210.563 \) billion. The survey CVs were 0.321 for sardine and 0.138 for anchovy (Mhlongo et al. 2012). The survey did not extend past Cape Infanta and thus no observation for the “south” sardine stock is available.

iv) The ratio of juvenile sardine to anchovy “in the sea” used in equation (A.26) is \( r_{2012} = 0.5 \times (0.0933 + 0.0458) \).

v) The model predicted recruitment in November 2011 is an inverse variance weighted average of the logarithms of two estimates (logarithms are taken as the distributions of the estimates themselves are assumed to be log-normal). The first estimate comes from the recruitment observed in the 2012 recruit survey:

\[ N^S_{j,2012,r} = \frac{1}{k^S_{j,r}} N_{obs,S}^{12,1} \] (being the best estimate from equation (A.31))

\[ N^A_{j,2012,r} = \frac{1}{k^A_{j,r}} N_{obs,A}^{12,1} \] (being the best estimate from equation (A.31))

\[ N^{S,pred}_{j,2011,0} = (N^S_{j,2012,r})^0.5 e^{(6+1.5)M^S_j/12} + C^{S}_{j,2012,obs}) e^{0.5(6+1.5)M^S_j/12} \] (from equation (A.34))

\[ N^{A,pred}_{j,2011,0} = (N^A_{j,2012,r})^0.5 e^{1.5 M^A_j/12} + C^{A}_{j,2012,obs}) e^{0.5 M^A_j} \] (from equation (A.34))

where \( C^{A}_{1,2012,obs} = 32.050 \) billion, and \( C^{S}_{1,2012,obs} = 0.013 \) billion being the juvenile anchovy and sardine catch, respectively from 1 November 2011 to the day before the recruit survey in June 2012. The standard errors associated with the logarithms of these estimates are:

\[ \hat{\sigma}^S_{1,2012,rec} = \sqrt{0.321^2 + (\phi^S_{ac})^2 + (\chi^S)^2} \]
\[
\bar{\sigma}^A_{1,2012,rec} = \sqrt{0.138^2 + (\lambda^i)^2}
\]

As no survey observation of the “south” sardine stock was available for May 2012, this calculation is only performed for anchovy and sardine single or “west” stocks. See ix) for the sardine “south” stock.

vi) The second estimate comes from the stock recruitment curve, but needs to take account of the serial correlation in residuals about this curve, and so depends on the residual estimated about this curve for November 2010. Thus:

\[
N^*,pred_{j,2011,0} = f(\hat{SSB}'_{j,2011,N}) e^{x_{j,2011,0,0} \sigma_{x,N}}
\]

with a standard error of the logarithm of this estimate being given by

\[
\tilde{\sigma}_j^{i,2011} = \sqrt{1 - (s_{j,rec}^{i,j})^2} \sigma_{x,N}
\]

vii) The inverse variance weighted average of the logarithms of these two estimates is then given by;

\[
\ln(N^*,pred_{j,2011,0}) = \frac{\ln(N^*,pred_{j,2011,0}) + \ln(N^*,pred_{j,2011,0})}{\tilde{\sigma}_j^{i,2011,2012,rec} + \tilde{\sigma}_j^{i,2011}}
\]

Note that this process is essentially shrinking the estimate provided by the survey towards the mean provided by the stock recruitment relationship (adjusted for serial correlation), and is used for anchovy and the sardine single or “west” stocks only.

viii) The recruitment residual in November 2011, required in the calculation of the recruitment residual in November 2012 (equation A.4), is obtained from equation (A.3) as follows:

\[
\varepsilon_{2011} = \ln \left( \frac{N_{j,2011,0}}{f(\hat{SSB}'_{j,2011,N})} \right) / \sigma_{x,N}
\]

As no survey observation is available for May 2012 for the “south” stock, the November 2011 recruitment is estimated from the stock recruitment curve, taking account of the serial correlation in residuals about this curve, i.e.:  

\[
N^*,pred_{j,2011,0} = f(\hat{SSB}'_{j,2011,N}) e^{x_{j,2011,0,0} \sigma_{x,N}}
\]

The recruitment residual in November 2011 for the “south” stock is thus:

\[
\varepsilon_{j,2011} = s_{j,2011,0} \varepsilon_{j,2011,0} + \sqrt{1 - s_{j,2011,0}^2} \sigma_{j,2011}
\]

External inputs into the MP testing framework

Some of the parameters required in the observation model were sampled from the posterior distributions of the underlying operating models (de Moor and Butterworth, 2013b,c,d). In addition, historic catches were used
in the calculation of single or “west” stock sardine bycatch to anchovy ratios used in the implementation model. These parameters are detailed in this section.

**Correlation in survey residuals**

The single or “west” stock sardine and anchovy November survey residuals are given by (i = S, A):

\[
\mathbf{e}_{y,\text{Nov}}^i = \ln \hat{B}_{1,i,y}^{1,y} - \ln(k_{1,i,y}^{1,y} \hat{B}_{1,i,y}^{1,y}), \quad y = 1984, \ldots, 2011
\]  

(A.35)

where

\[\hat{B}_{1,i,y}^{1,y}\] is the operating model estimate of historic November 1+ biomass (in thousands of tons) of stock \(j\) of species \(i\) (i = S, A) in year \(y\), drawn from posterior distributions estimated by the operating models (de Moor and Butterworth, 2013b,c,d).

The standard deviations of the residuals are given by (i = S, A):

\[
\sigma_{\text{Nov}}^i = \sqrt{\frac{\sum_{y=1984}^{2011} (\mathbf{e}_{y,\text{Nov}}^i)^2}{\sum_{y=1984}^{2011} 1}}.
\]  

(A.36)

The correlation in the residuals between the single or “west” stock sardine and anchovy November survey estimates is:

\[
\rho_{\text{Nov}} = \frac{\sum_{y=1984}^{2011} \mathbf{e}_{y,\text{Nov}}^S \mathbf{e}_{y,\text{Nov}}^A}{\sqrt{\sum_{y=1984}^{2011} \sigma_{\text{Nov}}^S \sigma_{\text{Nov}}^A}}.
\]  

(A.37)

Similarly, the single or “west” stock sardine and anchovy May recruit survey residuals are given by (i = S, A):

\[
\mathbf{e}_{y,\text{rec}}^i = \ln N_{1,i,y}^{1,y} - \ln(k_{1,i,y}^{1,y} N_{1,i,y}^{1,y}), \quad y = 1985, \ldots, 2011
\]  

(A.38)

where

\[N_{1,i,y}^{1,y}\] is the operating model estimate of historic May recruitment (in billions) of stock \(j\) of species \(i\) (i = S, A), at the time of the recruit survey in year \(y\), drawn from posterior distributions estimated by the operating models (de Moor and Butterworth, 2013b,c,d).

The standard deviations of the residuals are given by:

\[
\sigma_{\text{rec}}^i = \sqrt{\frac{\sum_{y=1985}^{2011} (\mathbf{e}_{y,\text{rec}}^i)^2}{\sum_{y=1985}^{2011} 1}}.
\]  

(A.39)

17 The sum is taken over all years for which a survey estimate of recruitment exists.
The correlation in the residuals between the single or “west” stock sardine and anchovy recruit survey estimates is:

\[
\rho_{rec} = \frac{\sum_{y=1985}^{2011} \varepsilon_{y,S} \varepsilon_{y,A} / (\sum_{y=1985}^{2011} \sigma_{rec,S}^2 \sigma_{rec,A}^2)^{1/2}}{20}.
\]

(A.40)

**Ratio of sardine bycatch to anchovy between January and May**

The ratio of sardine bycatch to anchovy in the commercial catches from January to May is needed to simulate the 0-year-old single stock or “west” stock sardine caught prior to the recruit survey (equation (A.10)). The relationship between the historical sardine bycatch to anchovy ratio in the catches from January to May, together with the stock assessment model prediction for the ratio of single stock or “west” stock sardine to anchovy November recruitment, is used to provide this ratio. Only the most recent 10 years data is used in the below equations as future catches are assumed to more closely simulate those over the past decade, rather than earlier periods when fishing patterns may have differed, particularly since the additional anchovy sub-season was only introduced in 1999. The constant of proportionality estimated and the associated time series of residuals are as follows:

\[
k_{Jan-May} = \exp\left\{\sum_{y=2002}^{2011} \ln\left( C_{y,Jan-May}^{S,byc} / C_{y,Jan-May}^{A} \right) - \ln\left( \hat{N}_{1,y-1,0}^{S} / \hat{N}_{1,y-1,0}^{A} \right) \right\} / \sum_{y=2002}^{2011} 1 \]

(A.41)

and

\[
\varepsilon_{y,Jan-May}' = \ln\left( C_{y,Jan-May}^{S,byc} / C_{y,Jan-May}^{A} \right) - \ln(k_{Jan-May} \hat{N}_{1,y-1,0}^{S} / \hat{N}_{1,y-1,0}^{A})
\]

\[ y = 2002,...,2011 \]

(A.42)

where

- \( C_{y,m} \) is the anchovy catch (in thousands of tons) from landings that have targeted anchovy during month(s) \( m (m = Jan, May, Jun, Jul, Aug, Sep, Oct, Dec) \) in year \( y \) (Table A3);
- \( C_{y,m}^{S,byc} \) is the associated sardine bycatch (in thousands of tons), assumed in the two stock hypothesis to be only from the “west” stock (Table A3); and
- \( \hat{N}_{j,y,0}^{i} \) is the model estimated number of recruits of stock \( j \) of species \( i \) (\( i = S, A \)) in November of year \( y \) (from which catches of 0-year-old sardine and anchovy are made in year \( y + 1 \)), drawn from posterior distributions estimated by the operating models (de Moor and Butterworth, 2013b,c,d).

The subset of years used is that for which the catch data and assessed recruitment estimates for both species are available. The standard deviation of the residuals is given by:

\[
\sigma_{Jan-May} = \frac{1}{\sum_{y=2002}^{2011} \varepsilon_{y,Jan-May}'}^{1/2} / \sum_{y=2002}^{2011} 1.
\]

(A.43)
Ratio of sardine bycatch to anchovy in the commercial fishery during May

For equation (A.17), the estimated constant of proportionality and the associated time series of residuals for the juvenile single stock or “west” stock sardine to anchovy ratio from the commercial catches during May are as follows:

$$k_m = \exp \left( \frac{\sum_{y=2002}^{2011} \left[ \ln \left( \frac{C_{y,m}^{byc}}{C_{y,m}^{A}} \right) - \ln \left( \frac{\hat{N}_{1,y,r}^{S}}{\hat{N}_{1,y,r}^{A}} \right) \right]}{\sum_{y=2002}^{2011} 1} \right)$$

(A.44)

and

$$\epsilon_{y,m}^* = \ln \left( \frac{C_{y,m}^{byc}}{C_{y,m}^{A}} \right) - \ln \left( \frac{k_m \hat{N}_{1,y,r}^{S}}{\hat{N}_{1,y,r}^{A}} \right), \ y = 2002, ..., 2011 \ and \ m = May. \quad (A.45)$$

The associated residual standard deviation is:

$$\sigma_m = \sqrt{\frac{\sum_{y=2002}^{2011} (\epsilon_{y,m}^*)^2}{\sum_{y=2002}^{2011} 1}}, \ m = May. \quad (A.46)$$

A correlation coefficient between the January to May and May residuals, for use in equation (A.18) above, is then calculated by:

$$\rho_m = \frac{\sum_{y=2002}^{2011} \epsilon_{y,m}^* \epsilon_{y,m}^{*'}}{\sqrt{\sum_{y=2002}^{2011} 1} \sigma_{m-1} \sigma_m}, \ for \ m = May \ and \ m - 1 = JanMay. \quad (A.47)$$
Table A1. Parameters sampled from the Bayesian posterior distributions of de Moor and Butterworth (2013b,c,d)

<table>
<thead>
<tr>
<th>Parameter sampled</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N^{S/A, pred}_{j, 2011, a}$</td>
<td>Operating model predicted numbers at age of sardine/anchovy in November 2011 (in billions)</td>
</tr>
<tr>
<td>$\varepsilon^{S/A}_{j, 2010}$</td>
<td>Operating model estimated sardine/anchovy recruitment residual in November 2010</td>
</tr>
<tr>
<td>$a_j^{S/A}$</td>
<td>Maximum deterministic sardine/anchovy recruitment (in billions)</td>
</tr>
<tr>
<td>$b_j^{S/A}$</td>
<td>Sardine/anchovy spawner biomass below which median recruitment declines linearly with this biomass (in thousands of tons)</td>
</tr>
<tr>
<td>$K_j^{S/A}$</td>
<td>Sardine/anchovy average pristine level (“carrying capacity”)</td>
</tr>
<tr>
<td>$\sigma_{j, r}^{S/A}$</td>
<td>Standard deviation in the sardine/anchovy recruitment residuals</td>
</tr>
<tr>
<td>$s_{j, cor}^{S/A}$</td>
<td>Sardine/anchovy recruitment serial correlation</td>
</tr>
<tr>
<td>$S_j^s$, $a = 1, \ldots, 4$</td>
<td>Sardine commercial selectivity at age</td>
</tr>
<tr>
<td>$k_{j, r}^{S/A}$</td>
<td>Multiplicative bias associated with the hydroacoustic survey estimate of sardine/anchovy recruitment</td>
</tr>
<tr>
<td>$\hat{B}_{j, y, N}$, $y = 1984, \ldots, 2011$</td>
<td>Operating model estimated sardine/anchovy November 1+ biomass (in thousands of tons)</td>
</tr>
<tr>
<td>$\hat{N}_{j, y, r}$, $y = 1985, \ldots, 2011$</td>
<td>Operating model estimated sardine/anchovy May recruitment (in billions)</td>
</tr>
<tr>
<td>$\hat{N}_{j, y, 0}$, $y = 1984, \ldots, 2010$</td>
<td>Operating model estimated sardine/anchovy recruitment in November (in billions)</td>
</tr>
<tr>
<td>$(\lambda_{j, N, r}^{S/A})^2$</td>
<td>Additional variance (over and above the survey sampling CV) associated with the November/recruit sardine/anchovy surveys</td>
</tr>
</tbody>
</table>
Table A2a. Average 1984 to 2011 weights-at-age (in grams) from the historic catches ($\overline{W}_{j,ac}$, $i = S, A$). As sardine catch weight-at-age is not directly available, the average over all years is taken from proxy-annual catch weights-at-age calculated as an average of the November survey weight at age $a$ in year $y - 1$ and weight-at-age $a + 1$ in year $y$.

<table>
<thead>
<tr>
<th>Sardine single stock</th>
<th>Sardine “west” stock</th>
<th>Sardine “east” stock</th>
<th>Anchovy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{w}_{1,0c}$</td>
<td>22.40</td>
<td>$\overline{w}_{1,0c}$</td>
<td>21.71</td>
</tr>
<tr>
<td>$\overline{w}_{1,1c}$</td>
<td>53.49</td>
<td>$\overline{w}_{1,1c}$</td>
<td>51.71</td>
</tr>
<tr>
<td>$\overline{w}_{1,2c}$</td>
<td>68.83</td>
<td>$\overline{w}_{1,2c}$</td>
<td>65.65</td>
</tr>
<tr>
<td>$\overline{w}_{1,3c}$</td>
<td>79.44</td>
<td>$\overline{w}_{1,3c}$</td>
<td>74.39</td>
</tr>
<tr>
<td>$\overline{w}_{1,4c}$</td>
<td>86.35</td>
<td>$\overline{w}_{1,4c}$</td>
<td>79.54</td>
</tr>
<tr>
<td>$\overline{w}_{1,5+c}$</td>
<td>89.01</td>
<td>$\overline{w}_{1,5+c}$</td>
<td>81.41</td>
</tr>
</tbody>
</table>

Table A2b. Average 1984 to 2011 weights-at-age (in grams) from the historic November spawner biomass surveys ($\overline{w}_{j,a}$, $i = S, A$).

<table>
<thead>
<tr>
<th>Sardine single stock</th>
<th>Sardine “west” stock</th>
<th>Sardine “east” stock</th>
<th>Anchovy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{w}_{1,1}$</td>
<td>44.81</td>
<td>$\overline{w}_{1,1}$</td>
<td>43.41</td>
</tr>
<tr>
<td>$\overline{w}_{1,2}$</td>
<td>62.89</td>
<td>$\overline{w}_{1,2}$</td>
<td>60.46</td>
</tr>
<tr>
<td>$\overline{w}_{1,3}$</td>
<td>75.79</td>
<td>$\overline{w}_{1,3}$</td>
<td>71.48</td>
</tr>
<tr>
<td>$\overline{w}_{1,4}$</td>
<td>84.32</td>
<td>$\overline{w}_{1,4}$</td>
<td>78.06</td>
</tr>
<tr>
<td>$\overline{w}_{1,5+}$</td>
<td>89.74</td>
<td>$\overline{w}_{1,5+}$</td>
<td>81.84</td>
</tr>
</tbody>
</table>
Table A3. Anchovy catch (in thousands of tons) from landings that have targeted *anchovy* ($C_{y,\text{janmay}}^A$), for five-month (“janmay”), five single month (“may”, “jun”, “jul”, “aug”, “sep”), and a three-month (“octdec”) periods, with the associated recorded landings of sardine bycatch ($C_{y,m}^S$, also in thousands of tons).

<table>
<thead>
<tr>
<th>Year</th>
<th>$C_{y,\text{janmay}}^A$</th>
<th>$C_{y,\text{may}}^A$</th>
<th>$C_{y,\text{jun}}^A$</th>
<th>$C_{y,\text{aug}}^A$</th>
<th>$C_{y,\text{sep}}^A$</th>
<th>$C_{y,\text{octdec}}^A$</th>
<th>$C_{y,\text{janmay}}^S$</th>
<th>$C_{y,\text{may}}^S$</th>
<th>$C_{y,\text{jun}}^S$</th>
<th>$C_{y,\text{aug}}^S$</th>
<th>$C_{y,\text{sep}}^S$</th>
<th>$C_{y,\text{octdec}}^S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1987</td>
<td>377.5</td>
<td>14.9</td>
<td>50.6</td>
<td>78.5</td>
<td>67.9</td>
<td>24.4</td>
<td>1.1</td>
<td>0.3</td>
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<td>1.0</td>
<td>0.2</td>
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<tr>
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<td>74.3</td>
<td>60.7</td>
<td>70.4</td>
<td>38.7</td>
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</tr>
<tr>
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<td>83.0</td>
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<td>13.7</td>
<td>#</td>
<td>#</td>
<td>5.1</td>
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<td>0.3</td>
<td>#</td>
<td>#</td>
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<td>42.9</td>
<td>10.8</td>
<td>0.9</td>
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<td>1.7</td>
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<td>27.2</td>
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<td>16.5</td>
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<td>1.2</td>
<td>2.8</td>
<td>1.2</td>
<td>#</td>
</tr>
</tbody>
</table>

* A landing is assumed to have targeted anchovy when the ratio anchovy : (anchovy + directed sardine + horse mackerel + round herring) exceeds 0.5 (in terms of mass).

* As no anchovy were landed during these months, sardine bycatch with anchovy is not applicable.
Figure A1. The historic ratio of \( \leq 14 \text{cm} \) sardine to \( >14 \text{cm} \) sardine in the directed sardine fishery. The two ratios above 7\%, shown as open diamonds, are fixed at 7\% in the distribution from which future samples are made (equations (A.11) and (A.16)).
Figure A2. The regressions of the ratio of small sardine bycatch : anchovy\(^\text{18}\) in the monthly commercial catch against that observed in the recruit survey, i.e. minimising
\[ \sum_{y=2002}^{2011} \left( \frac{C_{y,\text{byc}}^{S}}{C_{y,\text{m}}^{A}} - k_{m} \left( \frac{N_{1,y,\text{m}}^{S,\text{obs}}}{N_{1,y,\text{m}}^{A,\text{obs}}} \right) \right)^{2} \] w.r.t. \( k_{m} \).

The outliers of commercial ratio of 0.69 in October to December 2010 (shown as an open diamond) is removed, as this could have been biased by the mid-water trawl experiments which occurred during this time. The regression including this outlier is given by the dotted line.

\(^{18}\) For cases where anchovy is the most common species by mass in the landing
Figure A3. The regressions between observed survey CV and model predicted abundance for a) sardine single stock November, b) anchovy November, c) sardine “west” stock November, d) sardine “south” stock November, e) sardine single stock May, f) anchovy May surveys, g) sardine “west” stock May and h) sardine “south” stock May, for use in equations (A.29), (A.30), (A.32) and (A.33). In b) the outlier (333,0.17) was excluded from the regression.