



# An initial attempt at a sex-disaggregated assessment for the South African hake resource, fitting directly to age-length keys

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## INTRODUCTION

Geromont *et al.* (1995) estimated a female proportion in the south coast longline catches of 83%. Furthermore, there are very clear sex-specific differences in somatic growth for both *M. paradoxus* and *M. capensis*, in fact more so than between species (see Fig. 1). Routine application of age-length keys to obtain catch-at-age proportions is conducted without attention to sex-specific differences, but sex-differential growth means that larger sized of males are not well represented in the catch. This could confound estimates based on catch-at-age data developed from a sex-aggregated age-length key, which might consequently underrepresent the number of older hake present (and therefore affect the estimates of natural mortality).

This document presents a first attempt at modelling the sexes separately, which requires fitting directly to age-length keys (ALKs) and length frequencies (e.g. Punt *et al.* 2006).

## THE MODEL

The model is sex-specific; however, apart from sex-specific age-length keys, all the data available are sex-aggregated. Some assumptions have therefore to be made.

As it is not possible to estimate a sex-specific selectivity-at-age, it is rather assumed that the selectivity-at-length is the same for both males and females. The model therefore needs to keep track of the numbers in each length class. Rather than move to a length-based model, the option being pursued is to keep the model age-structured and convert numbers-at-age into numbers-at-length using a length-at-age distribution (assumed to remain constant over time).

The catches cannot be disaggregated by sex, and there seems no reason to suppose that there is any preferential targeting by sex; thus, we assume that the annual fishing mortality generated by each fleet is the same on both males and females.

Note: for ease of reading, the 'species' superscript has been omitted below.

The numbers-at-age are converted to numbers-at-length as follows:

$$N_{y,a,l}^g = N_{y,a}^g P_{a,l}^g \quad (1)$$

where  $P_{a,l}^g$  is the proportion of fish of age  $a$  and sex  $g$  that fall in the length group  $l$  (i.e.,  $\sum_l P_{a,l}^g = 1$  for all ages  $a$ ). The matrix  $A$  is calculated under the assumption that length-at-age is normally distributed about a mean given by the von Bertalanffy equation, i.e.:

$$L_a \sim N\left[L_\infty\left(1 - e^{-k(a-t_0)}\right); \theta_a^2\right] \quad (2)$$

where  $\theta_a$  is the standard deviation of length-at-age  $a$ , which is modelled as a function of the expected length at age  $a$ , i.e.:

$$\theta_a = \beta\left[L_\infty\left(1 - e^{-k(a-t_0)}\right)\right]^\gamma \quad (3)$$

Population dynamics:

$$N_{y+1,a+1,l}^g = \left(N_{y,a,l}^g e^{-M_a/2} - \sum_f C_{y,l}^{g,f}\right) e^{-M_a/2} \quad \text{for } 0 \leq a \leq m-2 \quad (4)$$

$$N_{y+1,m,l}^g = \left(N_{y,m-1,l}^g e^{-M_{m-1}/2} - \sum_f C_{y,l}^{g,f}\right) e^{-M_{m-1}/2} + \left(N_{y,m,l}^g e^{-M_m/2} - \sum_f C_{y,l}^{g,f}\right) e^{-M_m/2} \quad (5)$$

where

$N_{y,a,l}^g$  is the number of fish of sex  $g$ , age  $a$  and length  $l$  at the start of year  $y$ ,

$M_a$  denotes the natural mortality rate on fish of age  $a$  (assumed – for the moment - to be the same for males and females),

$C_{y,l}^{g,f}$  is the estimated number of hake of sex  $g$  and length  $l$  caught in year  $y$  by fleet  $f$ , and

$m$  is the maximum age considered (taken to be a plus-group).

A Beverton-Holt stock-recruitment relationship is assumed, with the recruitment ( $R_y^g$ ) dependent on the female component of the spawning biomass and assuming a 50:50 sex-split at recruitment.

$$R_y^g = 0.5 \frac{\alpha B_y^{sp,females}}{\beta + B_y^{sp,females}} e^{(\zeta_y - \sigma_R^2/2)} \quad (6)$$

where

$\alpha$  and  $\beta$  are spawning biomass-recruitment relationship parameters,

$\zeta_y$  reflects fluctuation about the expected recruitment for year  $y$ , which is assumed to be normally distributed with standard deviation  $\sigma_R$ ;

$B_y^{sp,g}$  is the spawning biomass of sex  $g$  at the start of year  $y$ , computed as:

$$B_y^{sp,g} = \sum_a \sum_l f_a w_l^g N_{y,a,l}^g \quad (7)$$

where

$w_l^g$  is the mass of fish of sex  $g$  and length  $l$  and  $f_a$  is the proportion of fish of age  $a$  that are mature.

Catch

$$C_{y,a,l}^{g,f} = N_{y,a,l}^g e^{-M_a/2} S_{y,l}^f F_y^f \quad (8)$$

where

$S_{y,l}^f$  is the commercial selectivity,

$F_y^f$  is the fished proportion of a fully selected length class, for fleet  $f$ , assumed to be the same for males and females.

Note:  $S$  and  $F$  are assumed to be independent of  $g$ .

The estimated sex-aggregated catches-at-length to be compared with observations of length frequencies are:

$$C_{y,l}^f = \sum_g \sum_a C_{y,a,l}^{g,f} \quad (9)$$

The (known) annual catch-by-mass of fleet  $f$  is given by:

$$C_{y,l}^f = \sum_g \sum_a \sum_l w_l^g C_{y,a,l}^{g,f} \quad (10)$$

So that:

$$F_y^f = C_y^f / \left( \sum_g \sum_a \sum_l N_{y,a,l}^g e^{-M_a/2} S_{y,l}^f \right) \quad (11)$$

The likelihood function

The model is fitted to CPUE and survey abundance indices, commercial and survey catch-at-length data, as well as to the stock-recruitment curve to estimate model parameters, as in the baseline assessment. The contributions by each of these to the negative log-likelihood are not repeated here.

For years for which ALKs are available, the baseline assessment is also fitted to commercial and survey catch-at-age data. Here however, the model is fitted to the data underlying the ALKs directly, so that catch-at-length are used throughout. The ALKs are the only data that are available in sex-disaggregated form. The contribution of the ALKs to the negative log-likelihood is as follows.

Under the assumption that fish are sampled randomly with respect to age within each length-class, the contribution to the negative log-likelihood for the ALK data (ignoring constants) is:

$$-\ln L^{ALK} = -\sum_i \sum_l \sum_a \left[ A_{i,l,a}^{obs} \ln(\rho_{i,l,a}) - A_{i,l,a}^{obs} \ln(A_{i,l,a}^{obs}) \right] \quad (12)$$

were

$A_{i,a,l}^{obs}$  is the observed number of fish of age  $a$  that fall in the length class  $l$ , for ALK  $i$  (a specific combination of survey (or commercial fleet), year, species and gender)

$\rho_{i,a,l}$  is the model estimate of  $A_{i,a,l}^{obs}$ , computed as:

$$\rho_{i,a,l} = W_{i,l} \frac{C_{i,l} A_{a,l}}{\sum_a C_{i,l} A_{a,l}} \quad (13)$$

where  $W_{i,l}$  is the number of fish in length class  $l$  that were aged for ALK  $i$ .

**DISCUSSION**

With the addition of the length dimension, the model now runs extremely slowly and results have not yet been obtained.

**REFERENCES**

Geromont HF, Butterworth DS, Japp D and Leslie RW. 1995. Preliminary assessment of longline experiments: south coast hake. Unpublished document, Marine and Coastal Management, South Africa. WG/11/95/D:H:28. 12pp.

Punt AE, Smith DC, Tuck GN and Methot RD. 2006. Including discard data in fisheries stock assessments: two case studies from south-eastern Australia. Fisheries Research 79: 239-250.

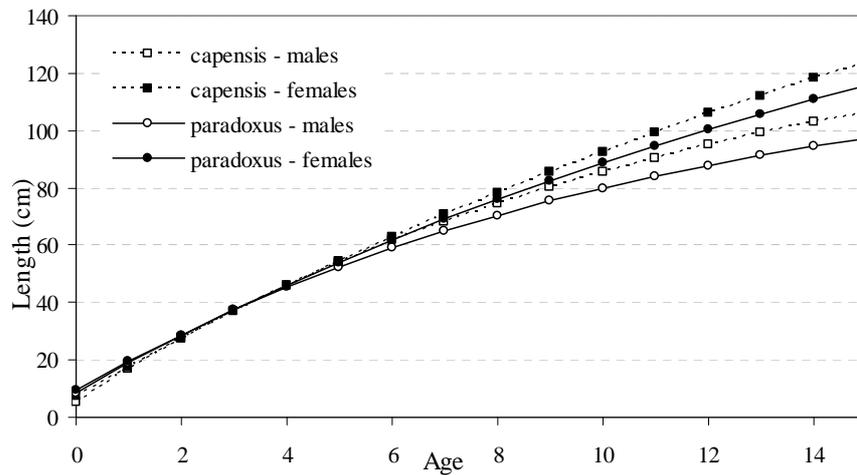


Fig. 1: Estimated mean length-at-age from the von Bertalanffi equation for males and females *M. capensis* and *M. paradoxus*